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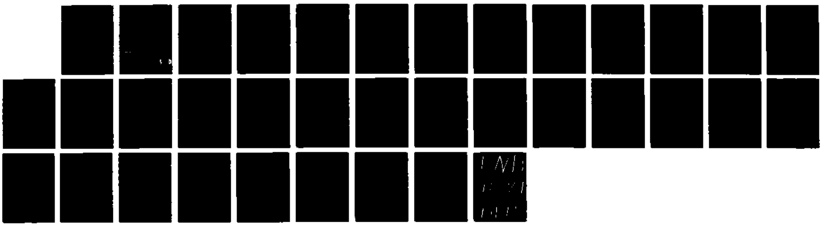

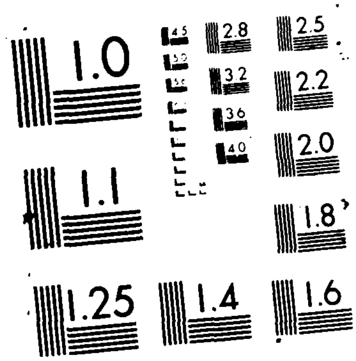
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ON THE APPROXIMATION OF THE OUTPUT PROCESS OF MULTI-USER RANDOM ACCESS COMMUNICATION NETWORKS

Submitted to:

Air Force Office of Scientific Research, NM
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June 1987

SCHOOL OF ENGINEERING AND APPLIED SCIENCE

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In this paper, the idea of approximating the output process of a slotted multi-user random access communication network by a process that depends on an underlying 3-state Markov chain and a stationary mapping rule, is introduced.

The mean time that a packet spends in a central node which is fed by packets originating from more than one network is used as a performance measure for the proposed approximation. As an example, a binary feedback limited sensing collision resolution algorithm is employed for each network, and the state and state transition probabilities for the corresponding Markov approximation are calculated.

The performance of the proposed approximation of the output process is compared with that of a 2-state Markov approximation and that of an independent packet departure process. Simulation results of the actual system are also provided.

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I. Introduction

A lot of work has been done towards the direction of developing communication protocols that determine how a single common resource can be efficiently shared by a large population of users. By now, it is well known that fixed assignment techniques are not appropriate for a system with large population of bursty users. In the latter case, random access protocols are more efficient and many of them have been suggested [1], [2]. Usually, the amount of information transmitted per time is of fixed length, called a packet. In most of the systems, time is divided into slots of length equal to the time needed for a packet transmission (slotted systems).

The deployment of an ever increasing number of multi-user random access communication networks, brought up the question of how packets whose destination is another network, should be handled. Thus, the issue of network interconnection or multi-hop packet transmission, arises, [3], [6], [7].

The basic problem in analyzing interconnected systems is that of characterizing the output process of a multi-user random access communication system; i.e., the departure process of the successfully transmitted packets. Another problem is how a random access protocol operates in the presence of a node that forwards exogenous traffic coming from other networks. The latter problem can be avoided by assigning a separate channel to the exogenous traffic. In this case, the operation of the system is not affected by the exogenous traffic but the problem of optimum allocation of the available resources (channels), arises. The latter issue has been discussed in [3], where the objective is to maximize the throughput of the interconnected networks. In [3], delay analysis was not performed and only simulation results were obtained.

The output process of a multi-user random access communication system depends on the protocol that has been deployed. Description of that process is a difficult task and only approximations based on special assumptions have been attempted, [4]-[7].

In this paper, the output process of a multi-user random access slotted communication system is approximated by a process which depends on an underlying 3-state Markov chain and a stationary mapping rule. A visit to a state takes place at the end of a slot and corresponds to one or zero departures, depending on the state and according to the stationary mapping. The state space of the Markov chain is given by

$$S = \{0, 1, 2\} = \{I, S, C\}$$

where I(0), S(1) and C(2), respectively, correspond to an idle, involved in a successful packet transmission or involved in a packet collision, slot. The feedback information available to the users at the end of each slot may not be ternary.

The motivation behind the adoption of the Markov model to approximate the output process is due to the fact that the output of the channel at the current time slot depends strongly on the outcome in the previous slot. This dependence is introduced by the collision resolution algorithm that is always present in a random access multi-user communication system. Thus, an independent packet departure model would be both unrealistic and intuitively not pleasing. On the other hand, a 2-state (arrival-no arrival) Markov model would also be inefficient since idle and collided slots would correspond to the same state. Clearly, the behavior of the system in a slot following a no-arrival slot depends strongly on whether the no-arrival slot was an idle or a collided one. The latter observation motivates the selection of the proposed 3-state Markov approximation of the output process.

In the next Section, the approximation model for the output process is introduced. The algorithm, which is used as an example in this paper for conflict resolution within the network, is also described in that section.

In section III, the steady state and state transition probabilities of the underlying Markov chain are calculated as the average state and state transition probabilities of the channel status, under the operation of the deployed random access algorithm.

In section IV, the queueing problem that appears in a central node which receives and retransmits packets originating from N independent multi-user random access slotted communication systems, is formulated. Then, a method is provided to calculate the mean time that a packet spends in the central node. The latter is used as a measure of the performance of the proposed approximation model of the output process.

In the last section, results for the average time that a packet spends in the central node are presented and conclusions are drawn. The performance of the proposed approximation of the output process is compared with that of a 2-state Markov approximation and that of an independent packet departure process. Simulation results of the actual system are also provided.

II. The approximation of the output process

We consider a system in which N independent multi-user random access synchronized slotted communication networks, are present. It is assumed that the length of a slot is the same for all networks of the system.

Let $\{\tilde{x}_i^j\}_{j\geq o}$ denote the process that describes the state of the i^{th} channel at the end of the slots. The state space of $\{\tilde{x}_i^j\}_{j\geq 0}$ is assumed to be given by

$$S^i = \{0,\ 1,\ 2\} = \{I,\ S,\ C\} \tag{1}$$

as it is explained in the Introduction. Since a packet appears in the output process if and only if it is the only one transmitted within the corresponding slot, the output process of the i^{th} network, $\{a_i^j\}_{j\geq 0}$, can be clearly described via the mapping

$$a_i(\tilde{x}_i^j) = \begin{cases} 1 & \text{if} \quad \tilde{x}_i^j = 1 \\ 0 & \text{if} \quad \tilde{x}_i^j = 0,\ 2\,. \end{cases} \tag{2}$$

The process $\{\tilde{x}_i^j\}_{j\geq 0}$ is controlled by the deployed random access algorithm and it is generally non-Markov. However, this process can be approximated by a Markov process $\{x_i^j\}_{j\geq 0}$ which has the same state space as $\{\tilde{x}_i^j\}_{j\geq 0}$ and is ergodic within the stability region of the random access algorithm. A Bernoulli model of the output process is unrealistic due to the dependence introduced by the collision resolution algorithm. A 2-state Markov model can also be inefficient for the reasons mentioned in the Introduction.

As an example, we consider multi-user random access slotted communication networks in which a binary (Collision/Non-Collision) feedback limited sensing collision resolution algorithm is deployed. This algorithm has been developed and analyzed in [11] and [10]. There, analysis was limited to the derivation of the maximum stable throughput and the average packet delay. The characterization of the process of the successfully transmitted packets, i.e. the output process of the network, is still an open problem.

A brief description of the collision resolution algorithm is provided at this point. Each user is assigned a counter whose initial value is zero (no packet to be transmitted).

This counter is updated according to the steps of the algorithm and the feedback from the channel. Upon packet arrival, the counter content increases to one. Users whose counter content is equal to one at the beginning of a slot, transmit in that slot. If the channel feedback is collision (C), the counters whose content is greater than one increase it by one; the counters whose content is one maintain this value with probability p (splitting probability) or increase it to two with probability 1-p. If the channel feedback is non-collision (NC), all non-zero counters decrease their content by one. A detailed description of the algorithm can be found in [10], [11].

To be able to apply the proposed 3-state Markov approximation of the output process of a network that operates under the described collision resolution algorithm, the steady state and state transition probabilities of the output process need to be calculated.

III. The steady state and state transition probabilities

Since the Markov model is only an approximation of the output process, it seems natural to estimate the steady state and state transition probabilities of the Markov chain by calculating the steady state probabilities that a particular state or state transition occurs in the output process, under stable operation of the network. This calculation is not always straightforward. The procedure to be followed depends on the class of random access algorithms which has been employed.

In this section, we calculate these probabilities for a network in which a binary (C/NC) feedback limited sensing random access algorithm has been deployed. The procedure that is followed can also be applied to a system in which some other limited sensing algorithm is deployed. In the latter case, the complexity of the calculations may be increased.

An important quantity in the analysis of the random access algorithm under consideration, is the session length. A technical definition of the session via the use of an imaginary marker, can be found in [12], [13]. In essence, a session is a sequence of consecutive slots that starts and ends at points in which the system regenerates itself. The multiplicity of a session is determined by the number of packet transmission attempts at the first slot of a session.

The following quantities are useful in the analysis that is presented in this section.

ij-slot : A slot that is in state i and it is followed by a slot in state j, $i, j \varepsilon \{I, S, C\}$.

i-slot : A slot that is in state i, $i \varepsilon \{I, S, C\}$.

internal slot : An ij-slot is internal if both slots belong to the same session; i-slots are always internal, $i, j \varepsilon \{I, S, C\}$.

l_k : Length of a session of multiplicity k (in slots).

L_k : Expected value of l_k.

L : Expected value of L_k with respect to k.

τ_k^{ij} (τ_k^i) : Number of internal ij-slots (i-slots) in a session of multiplicity k, $i, j \varepsilon \{I, S, C\}$.

T_k^{ij} (T_k^i) : Expected value of τ_k^{ij} (τ_k^i).

T^{ij} (T^i) : Expected value of T_k^{ij} (T_k^i) with respect to k.

t_n^{ij} (t_n^i) : Number of internal ij-slots (i-slots) in the n^{th} session from the time origin.

t_n : Length of the n^{th} session from the time origin (in slots).

q^i (q^{ij}) : Steady state probability of a slot in state i (steady state probability of a slot in state i followed by a slot in state j), i, j ε {I, S, C} .

p(i,j): Conditional steady state transition probability of having a j-slot in the next slot given that we have an i-slot in the current slot, i, j ε {I, S, C}.

The objective in this section is the calculation of the conditional and state probabilities p(i,j) and q^i , respectively, i, j ε {I, S, C}. Clearly, p(i,j) can be obtained by dividing the joint probability q^{ij} by the state probability q^i, i, j ε {I, S, C}. Thus, it suffices to calculate the probabilties q^{i}, q^{ij}, i, j ε {I, S, C}.

Under stable operation of the algorithm, the steady state probability of having a slot involved in a successful transmission, q^S, in the output process, is simply given by the cumulative input rate to the network, λ_i. It should be noted that the validity of the results that are obtained in this section is contingent upon the validity of the assumption that $\lambda_i < S_{i,\, max}$, where $S_{i,\, max}$ is the maximum stable throughput of the algorithm associated with the i^{th} network.

As it will become clear later, an important quantity for the calculation of the probabilities q^i,q^{ij}, i,j ε {I, S, C}, is the mean session length, L. The latter can be calculated by following procedures similar to those that appear in [11], [12], [13], [14]. In fact, for the specific algorithm under consideration, L has been calculated in [10] and [11], and numerical results can be found in [10]. For the completion of the calculations of this section we start by calculating L.

From the description of the algorithm the following equations can be written.

$$l_0 = 1 \quad , \quad l_1 = 1 \tag{3a}$$

$$l_k = 1 + l_{\phi_1 + F_1} + L_{k-\phi_1 + F_2} \quad , \quad k \geq 2. \tag{3b}$$

F_1 and F_2 are Poisson random variables over T=1 (length of a slot) with probability density $P_f(\,)$; ϕ_1 is Binomial with parameters k and p (p = .5) and probability density $b_k(\,)$. By considering the expected values in (3) with respect to all random variables involved, we obtain

$$L_0 = 1 \quad , \quad L_1 = 1 \tag{4a}$$

$$L_k = 1 + \sum_{F_1=0}^{\infty} \sum_{\phi_1=0}^{k} P_f(F_1)\, b_k(\phi_1)\, L_{\phi_1+F_1} + \sum_{F_2=0}^{\infty} \sum_{\phi_1=0}^{k} P_f(F_2)\, b_k(\phi_1) L_{k-\phi_1+F_2} \quad , \quad k \geq 2. \tag{4b}$$

The infinite dimensionality linear system of equations in (4) can be written in the general form

$$L_k = h_k + \sum_{j=0}^{\infty} a_{kj} L_j \quad , \quad k \geq 0. \tag{5}$$

The most widely used definition of stability is the one which relates it with the finiteness of L_k, for $k<\infty$. In [10], [11] it has been found that the system is stable for $\lambda_i < S_{i,\,max} = .36$ (packets/packet length). The authors in [10], [11] were actually able to find a (linear) upper bound on L_k which is finite for $k<\infty$. $S_{i,\,max}$ is then defined as the supremum over all rates λ_i for which such a bound, L_k^u, was possible to obtain.

The existence of $L_k^u < \infty$, for $k<\infty$, implies that (5) has a non-negative solution, L_k; the solution $\bar{L}_k$ of the finite dimentionality system of equations

$$\bar{L}_k = h_k + \sum_{j=0}^{J} a_{kj} \bar{L}_j \quad , \quad 0 \leq k \leq J \tag{6}$$

is a lower bound on L_k and $\bar{L}_k \rightarrow L_k$ as $J \rightarrow \infty$, [11], [14], [15].

It turns out that for sufficiently large J (e.g. 15), $\bar{L}_k$ is extremely close to L_k and thus, for practical purposes, L_k is considered to be equal to $\bar{L}_k$, especially for λ_i outside the neighborhood of $S_{i,\,max}$. (The latter can be shown by calculating a tight upper bound on L_k and observing that it almost coincides with $\bar{L}_k$, see [11], [13], [14] for the procedure).

By solving (6) with a_{kj} and h_k given by,

$$a_{0j} = a_{1j} = 0 \quad , \quad 0 \le j \le J \tag{7a}$$

$$a_{kj} = \sum_{\phi_1=0}^{\min\{j,k\}} P_f(j-\phi_1)\, b_k(\phi_1) + \sum_{\phi_1 = \max\{k-j,0\}}^{k} P_f(j-k+\phi_1)\, b_k(\phi_1) \quad , \quad 0 \le j \le J \tag{7b}$$

and

$$h_k = 1 \quad , \quad 0 \le k \le J \tag{8}$$

we calculate the mean session length of multiplicity k. Since the multiplicities of successive sessions are independent and identically distributed random variables, the mean session length, L, is calculated by averaging L_k over all k ; k is the number of arrivals in a slot from a Poisson process with intensity λ_i. In fact, the average for $k \le J$ is sufficient.

In the next three sub-sections we calculate the steady state, q^i, and joint, q^{ij}, probabilities, $i, j \in \{I, S, C\}$.

III-A: Calculation of q^I, q^{CI}, q^{CS}.

We start by calculating the joint probability q^{CI}; i.e, the probability that a slot is involved in a packet collision and the slot that follows is idle. By using the notation which was introduced at the beginning of this section, the following equations can be written.

$$\tau_0^{CI} = 0 \quad , \quad \tau_1^{CI} = 0 \tag{9a}$$

$$\tau_k^{CI} = 1_{\{\phi_1+F_1=0\}} + \tau_{\phi_1+F_1}^{CI} + \tau_{k-\phi_1+F_2}^{CI} \quad , \quad k \ge 2. \tag{9b}$$

The definition of the session implies that the last slot of a session cannot be involved in a collision. Thus, all CI-slots are internal slots and the equations in (9) take into consideration all possible CI-slots that may appear in the output process.

By considering the expected values in (9) with respect to all random variables involved, we obtain an infinite dimensionality linear system of equations with respect to T_k^{CI} which is of the form of the one in (5). A truncated, up to J, version of that system would be of the form of that in (6). In fact, the coefficients a_{kj}, $0 \le k \le J$, $0 \le j \le J$, are also given by (7), while the constants h_k, $0 \le k \le J$ are given by

$$h_0^{CI} = h_1^{CI} = 0 \quad , \quad h_k^{CI} = P_f(0)\, b_k(0) \ , \ 2 \le k \le J. \tag{10}$$

Since

$$T_k^{CI} \le L_k \tag{11}$$

and since $L_k < \infty$, for $k < \infty$ and λ_i inside the stability region of the algorithm, it is implied that the finite system of equations with respect to T_k^{CI}, $0 \le k \le J$, has a non-negative solution $\tilde{T}_k^{CI}$, $0 \le k \le J$, which converges to T_k^{CI} as $J \to \infty$. The comments about the rate of the convergence of $\tilde{L}_k$ are valid for $\tilde{T}_k^{CI}$ also. Thus, an extremely good approximation of the mean number of CI-slots in a session, T^{CI}, is obtained by averaging $\tilde{T}_k^{CI}$ over all $0 \le k \le J$.

By applying the law of large numbers we obtain the following expression for the joint probability q^{CI}.

$$q^{CI} = \lim_{M \to \infty} \frac{\sum_{n=1}^{M} t_n^{CI}}{\sum_{n=1}^{M} t_n} = \lim_{M \to \infty} \frac{\frac{1}{M} \sum_{n=1}^{M} t_n^{CI}}{\frac{1}{M} \sum_{n=1}^{M} t_n}. \tag{12}$$

Clearly, the random variables t_n^{CI}, $n \ge 1$, are independent and identically distributed. The same holds for t_n, $n \ge 1$. Since

$$E\{t_n^{CI}\} = T^{CI} < \infty \quad \text{and} \quad E\{t_n\} = L < \infty \tag{13}$$

by applying the strong law of large numbers to (12) we obtain, [18],

$$q^{CI} = \frac{T^{CI}}{L} \; . \tag{14}$$

To calculate the probabilities q^I and q^{CS} we follow exactly the same procedure. T_k^I and T_k^{CS} are computed by solving finite dimensionality linear systems of equations of the form of (6), whose coefficients are derived in Appendix A. Then, T^I and T^{CS} are computed as in the case of T^{CI}. Finally, the strong law of large numbers is again applied and the probabilities q^I and q^{CI} can be calculated by the following expressions.

$$q^I = \frac{T^I}{L} \quad , \quad q^{CS} = \frac{T^{CS}}{L}. \tag{15}$$

There is an easier way to calculate the steady state probability of having an idle slot, q^I, *for the particular algorithm. This procedure utilizes a property of the binary rooted* tree which can describe the splitting operation of the algorithm. According to that property, the number of terminal nodes is equal to the number of intermediate nodes plus one. Since the intermediate nodes correspond to collisions and the terminal nodes correspond to idle or successful slots, within a session, the following equation holds.

$$E + S - 1 = C$$

where E, S, C is the mean number of idle, successful or collided slots in a session. Since $S = \lambda_i L$ and

$$E + S + C = L$$

we can calculate q^I from the expression

$$q^I = \frac{E}{L} = \frac{1}{2} + \frac{1}{2L} - \lambda_i .$$

III-B: <u>Calculation of q^{IS} and q^{SI}</u>

We start by calculating the joint probability q^{IS}. The probability that the last slot of a session is idle will be needed and thus this quantity is calculated at first.

Let l_k^i be a random variable associated with the last slot of a session of multiplicity k that takes the following values.

$$l_k^i = \begin{cases} 0 & \text{if the slot is involved in a successful transmission} \\ 1 & \text{if the slot is idle.} \end{cases}$$

From the operation of the algorithm it can be observed that the following equations hold.

$$l_0^i = 1 \quad , \quad l_1^i = 0 \quad , \quad l_k^i = l_{k-\phi_1+F_2}^i \quad , \quad k \geq 2. \tag{16}$$

By considering the expected values in (16) we obtain the following infinite dimensionality system of linear equations

$$L_0^i = 1 \quad , \quad L_1^i = 0 \tag{17a}$$

$$L_k^i = \sum_{F_2=0}^{\infty} \sum_{\phi_1=0}^{k} P_f(F_2)\, b_k(\phi_1)\, L_{k-\phi_1+F_2}^i . \tag{17b}$$

Notice that L_k^i is the probability that the last slot of a session of multiplicity k is idle and thus $L_k^i \leq 1 < \infty$, for $k<\infty$. The system in (17) is of the form of that in (5). By using the same arguments as those used in the calculation of T^{CI} we can solve a truncated, up to J, version of (17) and obtain a very good approximation of L_k^i. By averaging the latter over all $k \leq J$, we can approximate L^i, the probability that the last slot of a session is idle.

For the internal IS-slots of the output process, the following equations hold.

$$\tau_0^{IS} = 0 \quad , \quad \tau_1^{IS} = 0 \tag{18a}$$

$$\tau_k^{IS} = \tau_{\phi_1+F_1}^{IS} + \tau_{k-\phi_1+F_2}^{IS} + 1_{\{l_{\phi_1+F_1}^i = 1 \ , \ k-\phi_1+F_2 = 1\}} . \tag{18b}$$

Notice that the idle slots which are the last of a session and are followed by a session of multiplicity 1 (that would give an IS-slot), are not considered by the expressions in (18).

By considering the expected values in (18), we obtain an infinite dimensionality system of linear equations with respect to T^{IS}. The comments that were made in the calculation of T^{CI} apply to this case again and thus T^{IS} can be calculated by solving a truncated version of the system in (18). The resulting finite dimensionality system is of the form of that in (6) with coefficients a_{kj}, $0 \le j \le J$, $0 \le k \le J$, given by (7) and constants given by

$$h_0^{IS} = 0 \quad , \quad h_1^{IS} = 0 \tag{19a}$$

$$h_k^{IS} = P_f(1)\, b_k\,(k) \sum_{F_1=0}^{J-k} P_f(F_1)\, L_{k+F_1}^{i} + P_f(0)\, b_k\,(k-1) \sum_{F_1=0}^{J+1-k} P_f(F_1)\, L_{k-1+F_1}^{i}\,. \tag{19b}$$

The average number of internal IS-slots in a session, T^{IS}, is then approximated by averaging T_k^{IS} over all $k \le J$.

By invoking the strong law of large numbers and the ergodic theorem for stationary processes, we prove in Appendix B that the joint probability q^{IS} is given by the following expression

$$q^{IS} = \frac{T^{IS}}{L} + \lambda_i e^{-\lambda_i} \frac{L^i}{L}. \tag{20}$$

The joint probability q^{SI} can be calculated by following a procedure similar to the one developed in the calculation of q^{IS}. The quantities and the equations that are involved in this calculation are given in Appendix C.

III-C: <u>Calculation of q^C, q^{CC}, q^{II}, q^{IC}, q^{SS}, and q^{SC}.</u>

The steady state probability q^C and the remaining joint probabilities are calculated from the following balance equations of the probability mass.

$$
\begin{aligned}
q^{C} &= 1 - q^{I} - q^{S} \\
q^{CC} &= q^{C} - q^{CI} - q^{CS} \\
q^{II} &= q^{I} - q^{SI} - q^{CI} \\
q^{SS} &= q^{S} - q^{IS} - q^{CS} \\
q^{IC} &= q^{I} - q^{II} - q^{IS} \\
q^{SC} &= q^{S} - q^{SI} - q^{SS}.
\end{aligned}
\tag{21}
$$

In this section, the steady state, q^i, and the joint, q^{ij}, probabilities have been calculated, $i, j \varepsilon \{I, S, C\}$. The transition probabilities P(i, j) of the Markov chain are calculated from the expression

$$p(i, j) = \frac{q^{ij}}{q^{i}} \quad , \quad i, j \varepsilon \{I, S, C\}.$$

IV. Performance measure for the approximation of the output process.

In this section, a system that consists of a central node which receives and retransmits packets originating from several random access communication systems, is considered (Fig. 1). Each input stream represents the output process from a multi-user random access slotted communication system; that is, the process of the successfully transmitted within the latter system packets.

A packet arrival is declared at the end of the slot in which the packet was successfully transmitted. Thus, the arrival process to each input line is a discrete process. The arrival points in all streams coincide; that is, the networks are assumed to be synchronized and all slots are of the same length.

The service time in the central node is constant and equal to one, which is assumed to be the length of a slot. This implies that arriving and departing, from the central node, packets have the same length. The first in-first out (FIFO) service policy is adopted. More than one arrivals (from different input streams) that occur at the same arrival point are served in a randomly chosen order. The buffer capacity of the central node is assumed to be infinite.

The objective in this section is to calculate the mean time that a packet spends in the central node (waiting time plus service time), under the assumption that the output process of a network is governed by an underlying 3-state Markov chain and a corresponding stationary mapping rule. The mean time is used as a performance measure of the proposed approximation model of the output process and it is compared with simulation results of the actual system in the next section.

The selection of the particular system to measure the performance of the proposed approximation, is due to the wide application that this system has. For example, such a system appears in a multi-hop environment, where the destination of packets originating from a multi-user random access network, is another network. Also, such a system may appear in a single hop environment. An example of the latter can be a packet radio environment in which more than one networks operating in the same or neighboring regions can use a common central node [19].

A discrete single server queueing system with finite number of independent input streams and per stream arrivals governed by an underlying finite-state Markov chain, has been analyzed in [18]. The system that is considered in this section is a special case of the general queueing system in [18]. The state space of the Markov chain and the stationary mapping are given by (1) and (2), respectively.

If the conditions

$$\lambda_i < S_{i,\,max} \;,\; 1 \le i \le N \quad \text{and} \quad \sum_{i=1}^{N} \lambda_i < 1$$

are satisfied, then the queueing system is stable. The average number of packets , Q, in the central node can be calculated as the sum of the solutions of 3^N linear equations, [18]. Then, the mean time that a packet spends in the central node, D, is given in conjunction with Little's formula, by the following expression.

$$D = \frac{Q}{\sum_{i=1}^{N} \lambda_i}. \tag{22}$$

Note that under stable operation of the networks ($\lambda_i < S_{i,\,max}$, $1 \le i \le N$), the adopted mapping rule in (1) implies that the input rate of the i^{th} stream to the central node is equal to the input rate to the corresponding network.

Let us denote by $\overline{x}$ and $\overline{y}$ the N-dimensional vectors that describe the states of the N Markov chains in two consecutive time slots, $\overline{x}, \overline{y}, \varepsilon\, \overline{S} = S^1 x\, S^2 x ... x S^N$. Let $p(j;\overline{y})$ denote the probability that there are j packets in the central node and that the N-dimensional Markov chain is in state $\overline{y}$, and let $P(z\,;\overline{y})$ be the corresponding generating function. Then, the average number of packets in the system, Q, is given by the sum of the solutions of 3^N linear equations, [18]. These equations are given by

$$\sum_{\nu=0}^{N} \sum_{\overline{x}\,\varepsilon\, F_\nu} [2(\nu-1)\, P'(1;\overline{x}) + (\nu-1)(\nu-2)\, P(1;\overline{x}) + 2(\nu-1)\, p(0;\overline{x})] = 0 \tag{23a}$$

and any $3^N - 1$ from the following:

$$P'(1;\overline{y}) = \sum_{\nu=0}^{N} \sum_{\overline{x}\,\varepsilon\, F_\nu} [(\nu-1)P(1;\overline{x}) + P'(1;\overline{x}) + p(0;\overline{x})]p(\overline{x},\overline{y}) \quad , \quad \overline{y}\,\varepsilon\,\overline{S}\,. \tag{23b}$$

The unknown quantities in (23) are $P'(1;\bar{y})$, $\bar{y} \in \bar{S}$; $P'(1;\bar{y})$ denotes the value of the derivative of $P(z;\bar{y})$ at $z=1$. The set F_v is given by

$$F_v = \left\{ \bar{x} = (x_1, \cdots x_N) \in \bar{S} \ : \ \sum_{i=1}^{N} a_i(x_i) = v \right\}.$$

Since the input streams to the central node are independent, we have that

$$p(\bar{x}, \bar{y}) = \prod_{i=1}^{N} p_i(x_i, y_i) \quad , \quad p(0;\bar{x}) = p_0 \prod_{i=1}^{N} \pi_i(x_i)$$

$$P(1;x) = \pi(\bar{x}) = \prod_{i=1}^{N} \pi_i(x_i)$$

where $\pi_i(x_i)$ and $p_i(x_i, y_i)$ are the steady state and state transition probabilities of the Markov chain associated with the i^{th} input stream and p_0 is the probability that the central node is empty. The latter is given by, [18],

$$p_0 = 1 - \sum_{i=1}^{N} \lambda_i .$$

By solving the 3^N linear equations that are given by (23) and summing up the solutions, the average number of packets in the central node, Q, is obtained. Then, the mean time that a packet spends in the system is given by (22).

V. Results and conclusions

In this section, the performance of the proposed approximation model of the output process is compared with the performance of the actual system and that of some other approximations. The mean time that a packet spends in a central node which receives and retransmits packets originating from N=2 and N=3 multi-user random access networks, is used as the performance measure.

We consider a multi-user random access slotted communication network in which the binary (C/NC) feedback limited sensing collision resolution algorithm, described in section II, is deployed. For this network, the output process is approximated as described in the Introduction. The steady state and state transition probabilities of the Markov chain are calculated according to the procedures developed in section III.

In the sequel, we consider systems of N=2 and N=3 networks, as those described above, that feed a central node. The mean time that a packet spends in the central node is calculated as it is described in the previous section. The results (in slots) are shown in Tables 1 and 2, together with the results obtained from the simulation of the actual system. It can be clearly observed that the results obtained under the proposed approximation of the output process are very close to those obtained from the simulation, especially for total input traffic rate to the central node less than .99.

In Table 3, the values of the steady state and state transition probabilities that were computed from the procedures developed in section III, are compared with the corresponding values obtained from the simulation of the actual system. The coincidence (up to the second decimal point) between the analytical and the simulation results, show that the estimation of those probabilities, by solving truncated systems, is extremely good. The difference between the results which appears for small λ and for the probabilities that involve collisions, is due to the fact that very few collisions appear for those values of λ and thus the simulation results are not reliable.

If the output process of a network is approximated by a process in which departing packets from the same network are independent, then the resulting queueing system in the central node has been studied and the mean time that a packet spends in the central node, D_I, is given by, [8],

$$D_I = \left[1 + \frac{\sum_{n=1}^{N} \sum_{m>n}^{N} \lambda_n \lambda_m}{(1 - \sum_{n=1}^{N} \lambda_n) \sum_{n=1}^{N} \lambda_n} \right]$$

The results for D_I and for N=2 and N=3 appear in Tables 1 and 2, respectively.

If the output process of a network is approximated by a 2-state Markov chain (arrival, no-arrival), then the resulting queueing system in the central node has been studied in [9] and the mean time that a packet spends in the central node, D_M, is given by

$$D_M = \left[1 + \frac{\sum_{n=1}^{N} \sum_{m>n}^{N} \lambda_n \lambda_m \left[1 + \frac{\gamma_n}{1-\gamma_n} + \frac{\gamma_m}{1-\gamma_m} \right]}{(1 - \sum_{n=1}^{N} \lambda_n) \sum_{n=1}^{N} \lambda_n} \right]$$

where $\gamma_n = P(1/1) - P(1/0)$ and where P(1/1) and P(1/0) denote conditional probability of arrival given arrival in the previous slot, and arrival given no-arrival in the previous slot, respectively. Clearly, D_M can be also obtained by solving 2^N equations, according to the procedures described in the previous section. The results for D_M and for N=2 and N=3 appear in Tables 1 and 2, respectively.

By examining the results in Tables 1 and 2, we conclude that when the total input traffic to the central node is less than .99 and the input traffic to each network is substantial ($\geq$ .25), then the proposed approximation is the best among all considered in this section. The proposed approximation performs also well for light traffic. The 2-state Markov approximation seems to perform better for light input traffic to each network. In this range of input traffic, the simulation results are not very reliable since arrivals and especially collisions are very rare. Furthermore, the differences among the results is so small that concllusions is not easy to be drawn. On the other hand, for substantial input traffic to each network, the 2-state Markov model fails completely. This was expected since the latter model fails to identify idle slots from collided and there is a substantial number of the latter in the output process.

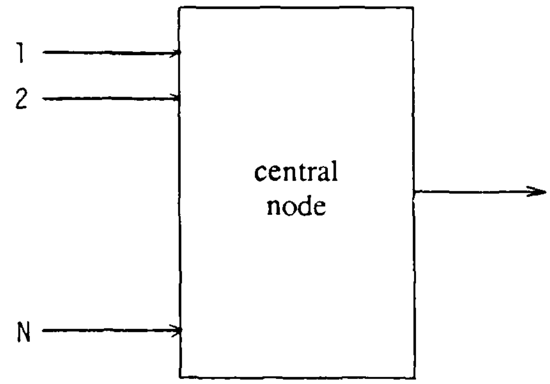

Figure 1.

λ	Indep.	2-Markov	3-Markov	Sim.	Net. Del.
.10	1.06	1.02	1.03	1.00	1.97
.20	1.08	1.17	1.09	1.02	3.33
.25	1.25	1.14	1.03	1.05	5.30
.30	1.37	1.23	1.10	1.12	11.38
.33	1.48	1.31	1.21	1.21	30.00
.35	1.58	1.41	1.31	1.30	87.70

Table 1.(N=2)

λ	Indep.	2-Markov	3-Markov	Sim.	Net. Del.
.10	1.14	1.05	1.06	1.02	1.97
.20	1.50	1.24	1.37	1.21	3.33
.25	2.00	1.55	1.91	1.70	5.30
.30	4.00	2.82	4.24	4.27	11.38
.31	5.43	3.74	5.94	6.25	15.00
.32	9.00	6.03	10.16	11.37	20.00
.33	34.00	22.34	39.87	48.89	30.00

Table 2.(N=3)

λ	.01	.10	.20	.30	.33
	Anal-Sim	Anal-Sim	Anal-Sim	Anal-Sim	Anal-Sim
q^S	.010-.010	.100-.100	.200-.200	.300-.300	.330-.330
q^I	*	.887-.887	.730-.731	.460-.461	.327-.329
q^C	**	.013-.013	.070-.069	.240-.239	.343-.341
q^{CI}	.246-.210	.215-.212	.184-.184	.156-.155	.148-.147
q^{CS}	.496-.530	.463-.466	.424-.424	.384-.385	.372-.373
q^{CC}	.258-.260	.322-.323	.392-.392	.460-.460	.480-.480
q^{SI}	.985-.985	.845-.846	.665-.668	.408-.410	.294-.297
q^{SS}	.015-.015	.138-.137	.256-.252	.359-.358	.390-.388
q^{SC}	***	.017-.017	.079-.079	.233-.232	.316-.315
q^{II}	.990-.990	.902-.902	.800-.799	.653-.653	.548-.550
q^{IS}	.009-.009	.090-.090	.164-.164	.217-.219	.225-.225
q^{IC}	****	.007-.007	.037-.036	.130-.128	.227-.225

* .9899-.9899 ** .0001-.0001
*** .00016-.00007 **** .000075-.00007

Table 3.

Appendix A

From the description of the algorithm, the following equations can be written.

$$\tau_0^I = 1 \quad , \quad \tau_1^I = 0$$

$$\tau_k^I = \tau_{\phi_1+F_1}^I + \tau_{k-\phi_1+F_2}^I \quad , \quad k \geq 2$$

and

$$\tau_0^{CS} = 0 \quad , \quad \tau_1^{CS} = 0$$

$$\tau_k^{CS} = 1_{\{\phi_1+F_1 = 1\}} + \tau_{\phi_1+F_1}^{CS} + \tau_{k-\phi_1+F_2}^{CS} \quad , \quad k \geq 2$$

where the random variables involved have been defined before. By considering the expectations in the above equations and by truncating the resulting system of linear equations, we obtain a finite systems of the form of (6) with coefficients gi.en by (7) and constants

$$h_0^I = 1 \quad , \quad h_k^I = 0 \quad , \quad k \geq 1$$

and

$$h_0^{CS} = 0 \quad , \quad h_1^{CS} = 0 \quad , \quad h_k^{CS} = P_f(0)\, b_k\,(1) + P_f(1)\, b_k(0) \quad , \quad k \geq 2.$$

Appendix B

In this Appendix we prove equation (20). Let us define the following random variables.

$$I_n^{IS} = \begin{cases} 1 & \text{if the } n^{th} \text{slot of a session is an IS–slot} \\ 0 & \text{otherwise} \end{cases}$$

$$J_v^{IS} = \sum_{n=1}^{N_v} I_n^{IS} \quad : \text{Number of IS–slots of the } v^{th} \text{ session}$$

$$N_v \quad : \text{Length of the } v^{th} \text{ session (in slots)}$$

$$\xi_\nu^{IS} = \begin{cases} 1 & \text{if the } \nu^{th} \text{ session has a non-internal IS-slot} \\ 0 & \text{otherwise} \end{cases}$$

Notice that ξ_ν^{IS} is associated with the last slot of the ν^{th} session. A non-internal slot is assigned to the session which its first slot belongs to.

The joint probability q^{IS} can be calculated, by using the law of large numbers, from the following expression.

$$q^{IS} = \lim_{M\to\infty} \frac{\sum_{\nu=1}^{M} J_\nu^{IS}}{\sum_{\nu=1}^{M} N_\nu}.$$

The above expression can be written as

$$q^{IS} = \lim_{M\to\infty} \frac{\sum_{\nu=1}^{M} (J_\nu^{IS} - \xi_\nu^{IS})}{\sum_{\nu=1}^{M} N_\nu} + \lim_{M\to\infty} \frac{\sum_{\nu=1}^{M} \xi_\nu^{IS}}{\sum_{\nu=1}^{M} N_\nu}$$

$$= \lim_{M\to\infty} \frac{\frac{1}{M}\sum_{\nu=1}^{M} t_\nu^{IS}}{\frac{1}{M}\sum_{\nu=1}^{M} N_\nu} + \lim_{M\to\infty} \frac{\frac{1}{M}\sum_{\nu=1}^{M} \xi_\nu^{IS}}{\frac{1}{M}\sum_{\nu=1}^{M} N_\nu}$$

Clearly, the random variables N_ν, t_ν^{IS} , $\nu\geq1$, are independent and identically distributed with mean value $L < \infty$ and $T^{IS} < \infty$ (since $T^{IS} < L$) , respectively. Thus, the strong law of large numbers asserts that

$$q^{IS} = \frac{T^{IS}}{L} + \frac{\lim_{M\to\infty} \frac{1}{M}\sum_{\nu=1}^{M} \xi_\nu^{IS}}{L} . \tag{B1}$$

The random variables ξ_ν^{IS} , $\nu \geq 1$, are not independent but $\{\xi_\nu^{IS}\}_{\nu\geq1}$ is a stationary process and ξ_ν^{IS} has expected value given by

$$E\{\xi_v^{IS}\} = P\{(\text{I–slot} \;,\; k'=1) \,/\, \text{last slot of a session}\}$$

where k' denotes the multiplicity of the next session. Since the two events are independent we can write that

$$E\{\xi_v^{IS}\} = L^i \lambda_1 e^{-\lambda_1} < \infty \;. \tag{B2}$$

The second term of the above product is the probability of having a session of multiplicity 1. By applying the ergodic theorem for stationary processess to (B1), [17], and by considering (B2), we obtain the expression in (20).

Appendix C

In this Appendix we present the procedure to calculate the joint probability q^{SI}. Let l_k^s, L_k^s and L^s be defined as the quantities l_k^i, L_k^i and L^i, respectively, by replacing the term idle by the term success. Since the last slot of a session is either idle or involved in a successful transmission we have that

$$L_k^s = 1 - L_k^i$$

$$L^s = 1 - L^i .$$

The equations that correspond to those in (18) are given by

$$\tau_0^{SI} = 0 \quad , \quad \tau_0^{SI} = 0$$

$$\tau_k^{SI} = \tau_{\phi_1+F_1}^{SI} + \tau_{k-\phi_1+F_2}^{SI} + 1_{\{l_{\phi_1+F_1}^s = 1 \,,\, k-\phi_1+F_2 = 0\}} \quad , \quad k \geq 2$$

and the equations that correspond to those in (19) are given by

$$h_0^{SI} = 0 \quad , \quad h_1^{SI} = 0$$

$$h_k^{SI} = P_f(0) b_k(0) \sum_{F_1=0}^{J-k} P_f(F_1)\, L_{k+F_1}^s \;.$$

Finally, q^{SI} is given by the following expression which is similar to that in (20).

$$q^{SI} = \frac{T^{SI}}{L} + e^{-\lambda}\frac{L^s}{L} .$$

T^{SI} is calculated in the same way as T^{IS}.

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